جامعة آل البيت دائرة ضمان الجودة والتخطيط

بسم الله الرحمن الرحيم

جامعة آل البيت

College of Science
Department of Mathematics
Course syllabus: Advanced Probability Theory
Second semester 2020/2021

1. Instructor Information:

Instructor Name	Ayat Al-Meanazel
Office Hours	Online Course
Office Number and Telephone Extension	57 Bukary
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2. Course Description:

Sets and Classes of Events: The event, algebra of sets, fields and σ-fields, classes of events; Random Variables: Functions and inverse functions, random variables, limits of random variables; Probability Space: Definition of probability, discrete probability space, general probability space, induced probability space; Distribution Functions: Decomposition of distribution functions, distribution functions of vector random variables, correspondence theorem; Expectation and Moments: Definition and properties of expectation, moments, inequalities; Convergence of Random Variables: Convergence in probability, convergence almost surely, convergence in distribution, convergence in r-th mean, convergence theorem for expectations, Fubini's theorem; Characteristics Functions: Definitions and properties, inversion formula, characteristic functions and moments, Bochner's theorem; Convergence of Distribution Functions: Weak convergence, Convergence of distribution functions and characteristics functions, convergence of moments; Independence: Definitions, multiplication properties, Zero-One laws; Laws of Large Numbers: Convergence of a series of independent random variables, Kolmogrov inequalities and almost sure convergence, stability of independent random variables; Central Limit Theorem: I.I.D. case, variable distributions, multivariate central theorem; Conditioning: Randon-Nikodym theorem, conditional expectation, properties of conditional expectation, martingales. If time permits and depending on the interest of the students Finite Markov Chain might be covered as well.

3. Course Information:

Course number: 401731	Course Title: Advanced Probability Theory	Level : Graduate (Master's Level)
Course Nature: Theoretical	Prerequisite: None	Lecture time: Wednesday 13:00-16:00
Academic year: 2020 – 2021	Semester: Second	Credit Hours: 3

4. Course Objectives:

This is a graduate course in probability theory designed for Master's level students in mathematics. The course covers core topics in measure theoretic probability and modern stochastic calculus. Brief review of basic probability concepts in a measure theoretic setting: probability spaces, random variables, expected value, conditional probability and expectation, independence, construction of probability spaces with emphasis on stochastic processes. Operator methods in probability: generating functions, moment generating functions, Laplace transforms, and characteristic functions. Notions of convergence: convergence in probability and weak laws of large numbers, convergence almost surely and strong laws of large numbers, convergence of probability measures and central limit theorems.

5. Intended Student Learning Outcomes:

Successful completion of the course should lead to the following outcomes:

1. **Knowledge and Understanding Skills:** Student expected to define and apply the concepts of sample space, events, probability, random variables and their distributions, expectation, variance and covariance of random variables. Understand theorems and inequalities concerning functions of random variables and the moment-generating functions.

2.	**Intellectual Analytical and Cognitive Skills:** Student expected to apply theorems and inequalities concerning functions of random variables and the moment-generating functions, Fubini's theorem, Bochner's theorem, Central Limit Theorem, multivariate central theorem, and Randon-Nikodym theorem.
3.	**Subject- Specific Skills:** Student expected to formulate and apply the definitions of convergence in distribution and in probability, weak and strong laws of large numbers, and central limit theorem.
4.	**Creativity /Transferable Key Skills/Evaluation:** Student expected to use statistical formulas and statistical structures.

6. Course Content:

Week	Chapter	Subject	Pages and Assignments
1+2	Chapter 1 Sets and Classes of events	1.1 The Event 1.2 Algebra of Sets 1.3 Fields and σ-Fields 1.4 Classes of Events	21-38 Odd Problems Page 39-41
2+3	Chapter 2 Random Variables	2.1 Functions and Inverse Functions 2.2 Random Variables 2.3 Limits of Random Variables	42-56 Odd Problems Page 63-65
4+5+6	Chapter 3 Probability Space	3.1 Definition of Probability 3.2 Some Simple Properties 3.3 Discrete Probability Space 3.4 General Probability Space 3.5 Induced Probability Space	66-84 Odd Problems Page 92-95
6+7	Chapter 4 Distribution Functions	4.1 Distribution Function of a Random Variable 4.2 Decomposition of Distribution Functions 4.3 Distribution Functions of Vector Random Variables 4.4 Correspondence Theorem	96-105 Odd Problems Page 107-110
8	Chapter 5 Expectation and Moments	5.1 Definition of Expectation 5.2 Properties of Expectation 5.3 Moments, Inequalities	111-125 Odd Problems Page 133-139
9+10	Chapter 6 Convergence of Random Variables	6.1 Convergence in Probability 6.2 Convergence Almost Surely 6.3 Convergence in Distribution 6.4 Convergence in r-th Mean 6.5 Convergence Theorem for expectations 6.6 Fubini's Theorem	140-161 Odd Problems Page 165-169
11+12	Chapter 7 Characteristics Functions	7.1 Definitions and Simple Properties 7.2 Some Simple Properties 7.3 Inversion Formula 7.4 Characteristic Functions and Moments 7.5 Bochner's Theorem	170-191 Odd Problems Page 193-197
12	Chapter 8 Convergence of Distribution Functions	8.1 Weak Convergence 8.2 Convergence of Distribution Functions and Characteristics Functions 8.3 Convergence of Moments	198-207 Odd Problems Page 209-212
13	Chapter 9 Independence	9.1 Definitions 9.2 Multiplication Properties 9.3 Zero-One laws	213-223 Odd Problems Page 227-230
13	Chapter 10 Laws of Large Numbers	10.1 Convergence of a Series of Independent Random Variables 10.2 Kolmogrov Inequalities and A.S. Convergence 10.3 Stability of Independent Random Variables	231-239 Odd Problems Page 250-255
14	Chapter 11 Central Limit Theorem	11.1 Introduction 11.2 I.I.D. Case 11.3 Variable Distributions 11.4 Multivariate Central Theorem	256-267 Odd Problems Page 269-275
14	Chapter 11 Conditioning	12.1 Randon-Nikodym Theorem 12.2 Conditional Expectation 12.3 Properties of Conditional Expectation 12.4 Martingales	276-289 Odd Problems Page 292-294

7. Teaching and learning Strategies and Evaluation Methods:

Learning Outcomes	Teaching Strategies	learning Strategies	Evaluation Methods
1) Learn the concepts of sets and classes of events, random variables, expectations, moments, characteristics functions 2) Construct Probability space.	- Writing on the blackboard - Ask students questions and discuss them - Solve various issues	Give: * homework * assignments	- Classroom presentations - Discussion - Midterm exam
1) Learn the concepts of independence Zero-One laws, laws of large numbers, central limit theorem, and conditioning. 2) Apply the definitions of convergence in distribution and in probability, weak and strong laws of large numbers and central limit theorem.	- Writing on the blackboard - Ask students questions and discuss them - Solve various issues	Give: * homework * assignments	- Classroom presentations - Discussion - Final exam

8. Assessment:

Assessment	Grade Proportion	Week/Dates
Midterm Exam	**30 %**	**13th Week**
Class Work (Quizzes, Homework and Attendance of the lecture)	**20 %**	
Final Exam	**50 %**	**End of Semester**
Total	**100 %**	

9. Text Book:

The main reference	Modern Probability Theory
Author(s)	B. Ramdas Bhat
Publisher	New Age International (P) Ltd.
Year	1999 (Reprint in 2004)
The edition	3rd edition
The reference website	https://www.amazon.fr/s?k=9788122411898&i=stripbooks&linkCode=qs

10. References and additional resources:

1)	Probability: Theory and Examples; Durrett, Rick. (4th ed.) Cambridge University Press, 2010.
2)	Measure Theory and Probability Theory; Krishna B. Athreya, Soumendra N. Lahiri (1st ed.) Springer Science and Business Media, 2006.
3)	Probability: A Graduate Course 3; AllenGut (1st ed.) Springer Science and Business Media, 2005.
4)	Probability and Measure Theory Ash, R.B. and C.A. (2nd ed.) 2000.