College of Science
Department of Mathematics
Course syllabus: Abstract Algebra (2)
First semester 2019/2020

1. Instructor Information:


## 2. Course Description:

As stated in the approved study plan.
Rings, subrings, integral domains, factor rings and ideals. Ring homomorphisms; polynomial rings; factorization of polynomials; reducibility and irreducibility tests; divisibility in integral domains; principal ideal domains and unique factorization domains.
3. Course Information:

| Course number: 401442 | Course Title: Abstract Algebra 2 | Level : Four year |
| :--- | :--- | :--- |
| Course Nature: Theoretical | Prerequisite: 0401342 | Lecture time: Sun. Tue. <br> Thu. 12:00 $-1: 00$ |
| Academic year: $2019-2020$ | Semester: First | Credit Hours: 3 |

## 4. Course Objectives

1. In abstract algebra I, students learn about groups and many of their properties. A group is a set with one operation and this operation satisfies certain conditions. In many important groups, there is another operation that we do not talk about in group theory. For instant numbers (Integers, rational numbers, $\cdots$ ), polynomials, functions, and matrices are structures with two operations, namely addition and multiplication. When considering these sets as groups, we simply used addition and ignored multiplication. In many cases, however, one want to take into account both addition and multiplication. The concept of rings does this, where a ring is a set endowed with two binary operations (addition and multiplication) and these two operations satisfy certain conditions. Students study rings and their properties and look at various examples of rings.
2. Commutative rings with unity where cancellation law holds are an abstraction of the Integers. These rings are called integral domains. Student will learn about integral domains and their properties. Also, students examine several examples of integral domains.
3. The concept of ideals and factor rings are introduced. Ideals are used to construct and study sophisticated factor rings. Maximal and prime ideals are introduced and used to construct fields and integral domains.
4. Homomorphisms are functions between rings that preserve the ring operations. One way to find out information about a ring is to study its interaction with other rings by way of
homomorphisms. Students learn a formal method for determining whether two rings are really the same through ring isomorphism.
5. Students are most familiar and most comfortable with polynomials. They have seen polynomials with realcoefficients (or integer coefficients) in college algebra and calculus classes and they worked on them as functions. As an abstraction, one can define polynomials with coefficients from commutative rings. These abstract structures are rings and are called polynomial rings. Students examine polynomial rings and study their properties.
6. In college algebra and calculus, students spend some time in finding zeros of polynomials and in factoring polynomials. In many cases, it is not easy to decide if a certain polynomial can be factored. Students learn some tests that enable them to decide the irreducibility of certain polynomials in more abstract setting. Also, students will see how these irreducible polynomials can be used to construct finite fields.
7. Students are expected to spend two to three hours, after each class, reading the material given in class. They should be able to redo the proofs of the theorems. A collection of exercises is assigned at the end of each chapter. Students are expected to work on these problems by themselves. They can discuss their ideas and solutions of these problems with their peers. In case, students were unable to solve a certain problem they can ask about it in class. I will ask students to present their work on some of these problems in class.

## 5- Intended Student Learning Outcomes

Successful completion of the course should lead to the following outcomes:
A. Knowledge and Understanding Skills: Student is expected to

A1. Learn and apply the elementary theorems and proof techniques of rings.
A2. Define, interpret and analyze fundamental principles and theory concerning subrings, ideals, principle ideals, prime ideals, maximal ideals, quotient rings, Boolean rings and direct sum of rings.
A3. Determine, use and apply homomorphisms and isomorphisms between rings.
B. Intellectual Analytical and Cognitive Skills: Student is expected to

B1. Define, interpret and analyze fundamental principles and theory concerning the basic algebraic structures of rings, integral domains, and fields.
B2. Demonstrate the ability to do direct proofs, proofs by contradiction, proof by contrapositive and proof by induction concerning properties of rings.
C. Subject- Specific Skills: Student is expected to

C1. Exhibit knowledge of reducibility tests, their usefulness, and apply the tests to determine the reducibility or irreducibility of a polynomial.
C2. Do factorization in an integral domains with a special emphasis on Unique Factorization Domains, Principal Ideal Domains and Euclidian Domains.
C3. Deal and examine polynomial rings. Demonstrate the division algorithm for polynomial over fields and its Applications.
D. Creativity /Transferable Key Skills/Evaluation: Student is expected to

D1. Read write and criticize proofs.
D2. Perform logical thinking.
D3. Using mathematical reasoning.

## 6- Course Content

| الموضـوع | الأسبوع |
| :---: | :---: |
| Introduction to Rings: Definition and Examples, Properties of Rings, Subrings. | 1+2 |
| Integral Domains: Definition and Examples, Fields, Characteristic of a Ring. | $3+4$ |
| Ideals and Factor Rings: Ideals, Factor Rings, Prime Ideals and Maximal Ideals. | 5+6 |
| First Exam |  |
| Ring Homomorphisms: Definition and Examples, Properties of Ring Homomorphisms, The Field of Quotients. | 7+8 |
| Polynomial Rings: Notation and Terminology, The Division Algorithm and Consequences. | $9+10$ |
| Factorization of Polynomials: Reducibility Tests, Irreducibility Tests, Unique Factorization in Z[x]. | $11+12$ |
| Second Exam |  |
| Divisibility and Integral Domains : Irreducibles and Primes, Unique Factorization Domains, Euclidian | $13+14$ |
| Extension Fields and Finite Fields | 15 |
| Final Exam | 16 |

## 7. Teaching and learning Strategies and Evaluation Methods:

| Learning Outcomes | Teaching <br> Strategies | learning <br> Strategies | Evaluation <br> Methods |
| :---: | :---: | :---: | :---: |
| A1. Learn and apply the elementary theorems and proof techniques of rings. A2. Define, interpret and analyze fundamental principles and theory concerning subrings, ideals, principle ideals, prime ideals, maximal ideals, quotient rings, Boolean rings and direct sum of rings. <br> A3. Determine, use and apply homomorphisms and isomorphisms between rings. | - Writing on the blackboard <br> - Ask students <br> questions and discuss them <br> - Solve various issues | Give homework assignments | - Classroom presentations <br> - Discussion <br> - First exam |
| B1. Define, interpret and analyze fundamental principles and theory concerning the basic algebraic structures of rings, integral domains, and fields. <br> B2. Demonstrate the ability to do direct proofs, proofs by contradiction, proof by contrapositive and proof by | - Writing on the blackboard <br> - Ask students questions and discuss them <br> - Solve various issues | Give homework assignments | - Classroom presentations - Discussion - Second exam |


| induction concerning properties of <br> rings. |  |  |  |
| :---: | :---: | :---: | :---: |
| C1. Exhibit knowledge of reducibility tests, <br> their usefulness, and apply the tests to <br> determine the reducibility or irreducibility <br> of a polynomial. |  |  |  |
| C2. Do factorization in an integral <br> domains with a special emphasis on <br> Unique Factorization Domains, | - Writing on the <br> blackboard <br> - Ask students <br> questions and discuss <br> them | Give <br> homework <br> assignments |  |
| Domains. <br> C3. Deal and examine polynomial Domains and Euclidian <br> rings. Demonstrate the division <br> algorithm for polynomial over fields <br> and its Applications. | - Solve various issues |  |  |
| D1. Read write and criticize proofs. | - Writing on the <br> blackboard <br> - Ask students <br> D2. Perform logical thinking. | Give <br> D3. Using mathematical reasoning. <br> them discuss <br> homework <br> assignments | - Classroom <br> presentations <br> - Discussion <br> - Final exam |

## 8- Assessment:

| Assessment | Grade <br> Proportion | Week/Dates |
| :---: | :---: | :---: |
| Class Work (Quizzes, Homework <br> and Attendance of the lecture) |  |  |
| First exam | $25 \%$ | $\mathbf{7}^{\text {th }}$ Week |
| Second exam | $25 \%$ | $\mathbf{1 2}^{\text {th }}$ Week |
| Final exam | $50 \%$ | End of <br> Semester |
| Total | $100 \%$ |  |

## 9. Text Book:

| The main reference | Contemporary Abstract Algebra |
| :--- | :--- |
| Author(s) | Joseph A. Gallian |
| Publisher | BROOKS/COLE |
| Year | $\mathbf{2 0 1 3}$ |
| The edition | EIGHTH EDITION |
| The reference <br> website | https://isidore.co/calibre/get/pdf/4975 |

## 10. References and additional resources:

| 1) | A First Course in Abstract Algebra by J. Fraleigh. |
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| 2) | David S. Dummit and Richard M. Foote. Abstract Algebra, |
| 3) | I. N. Herstein Topics in Algebra,. |

